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Proper fluid loading of thin shell theories and the prediction of pseudo-Stoneley resonances

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ABSTRACT

By including the effect of fluid loading for the thin spherical shell in a proper manner, so-called shell theories can predict the water borne pseudo-Stoneley waves described extensively in the literature. Shell theories give reasonably good results for the motion of a bounded elastic shell by using the assumption that various parts of the shell move together in some reasonable manner. Without proper fluid loading, however, shell theories do not predict the pseudo-Stoneley resonances observed in nature and predicted by exact theory. With proper fluid loading, as well as rotary inertia and translational and rotary kinetic energy terms, a shell theory can exactly predict these water borne resonances. These resonances are predicted by the shell theory and compared with results from exact elastodynamical calculations.

1. INTRODUCTION

Interest in sound scattering from submerged elastic shells is broadly based. For example, the structural engineering community has a considerable interest in the area of nondestructive testing via ultrasonic scattering, while in the area of remote sensing or object identification scattering from objects proves invaluable. The acoustical scattering from elastic objects or more simply the generation of resonances from bounded elastic shells can be described by acoustic and elastodynamic theory. Exact solutions for scattering from elastic targets exist for target shapes for which the elastodynamic equation is separable. For three dimensional targets separability is only possible for spherical and rectangular targets. For more complicated targets numerical techniques must be used. For axisymmetric smooth targets the extended boundary method of Waterman has proven useful though limited and an approximate theory would be desirable for more complicated targets. The purpose of this work is to develop an approximate theory based on shell theory that would be of use for general shapes that could also include structural loading. Shell theories afford a very powerful methodology to "build in" structural features usually via a variational principle in which some Lagrangian or Hamiltonian is constructed by introducing physical features. The simplest theories which always assume correlated motion of the inner and outer surfaces of the target include kinematic features and potential energy terms based on the generalized Hooke's law which leads to the lowest order symmetric mode. One can add rotational inertia to the Hamiltonian which allows an antisymmetric mode. One goes on from there to include higher order corrections. Further, proper fluid loading -- which from our experience is no easy task -- must be introduced correctly. The proper inclusion of fluid loading introduces a fluid borne wave: the pseudo-Stoneley wave. Clearly thin shell theories are rather geometrical and must be constructed for each shape due to their dependence on shape dependent dynamic factors. Non-spherical three dimensional objects present a problem which we seek to address in the future. In this work we wish to show the development and test the predictions of a suitable shell theory for spherical shells that can be readily generalized to spheroids and finite cylinders, and specifically we wish to show that proper fluid loading in an shell theory does allow the prediction of water-borne pseudo-Stoneley waves. In the following section we outline the derivation of a shell theory with many of the featured described. The theory is then used to compare with the exact method. and specifically the exact theory calculations of the water-borne wave mode. The results are then discussed and future work is described.

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2. DERIVATION OF EQUATIONS OF MOTION

In spherical shells membrane stresses (proportional to β) predominate over flexural stresses (proportional to β^2) where

$$\beta = \frac{1}{\sqrt{12}}\frac{h}{a}. \tag{1}$$

We differ from the standard derivation for the sphere[1] by retaining all terms of order β^2 in both the kinetic and potential energy parts of the Lagrangian and by considering the resonance frequencies for the fluid loaded case to be complex. We note that this level of approximation will allow us to include the effects of rotary inertia in our shell theory, as well as damping by fluid loading. The parameter β itself is proportional to the radius of gyration of a differential element of the shell and arises from integration through the thickness of the shell in a radial direction. We will use an implicit harmonic time variation of the form $\exp(-i\omega t)$. We begin our derivation by considering a u,v,w axis system on the middle surface of a spherical shell of radius a (measured to mid-shell) with thickness h, as shown in Fig. 1.

2.1. Lagrangian Variational Analysis

Our Lagrangian, L, is

$$L = T - V + W, \tag{2}$$

where T is the kinetic energy, V is the potential energy, and W is the work due to the pressure at the surface. The kinetic energy is given by

$$T = \frac{1}{2}\rho_s \int_0^{2\pi}\int_0^{\pi}\int_{-h/2}^{h/2}(\dot{u}_s^2 + \dot{w}_s^2)(a+x)^2 \sin\theta\, dx\, d\theta\, d\phi, \tag{3}$$

where the surface displacements are taken to be linear:

$$\dot{u}_s = (1+\frac{x}{a})\dot{u} - \frac{x}{a}\frac{\partial \dot{w}}{\partial \theta}, \tag{4}$$

and

$$\dot{w}_s = \dot{w}. \tag{5}$$

The motion of the spherical shell is axisymmetric since the sound field is torsionless. Thus there is no motion in the v-direction. Substitution of Eqs. (4) and (5) into Eq. (3) yields, after integration over x and ϕ,

$$T = \pi\rho_s \int_0^{\pi} \sin\theta[(\frac{h^5}{80a^2} + \frac{h^3}{2} + ha^2)\dot{u}^2 - 2(\frac{h^5}{80a^2} + \frac{h^3}{4})\dot{u}\frac{\partial \dot{w}}{\partial \theta} + (\frac{h^5}{80a^2} + \frac{h^3}{12})(\frac{\partial \dot{w}}{\partial \theta})^2 + (\frac{h^3}{12} + ha^2)\dot{w}^2]d\theta, \tag{6}$$

or, in terms of β,

$$T = \pi\rho_s h a^2 \int_0^{\pi} [(1.8\beta^4 + 6\beta^2 + 1)\dot{u}^2 - (3.6\beta^4 + 6\beta^2)\dot{u}\frac{\partial \dot{w}}{\partial \theta} + (1.8\beta^4 + \beta^2)(\frac{\partial \dot{w}}{\partial \theta})^2 + (\beta^2 + 1)\dot{w}^2]\sin\theta d\theta, \quad (7)$$

where the first and last terms in square brackets in Eq. (7) are associated with linear translational kinetic energies and the middle two terms are associated with rotational kinetic energies of an element of the shell.

The potential energy of the shell is

$$V = \frac{1}{2}\int_0^{\pi}\int_0^{2\pi}\int_{-h/2}^{h/2}(\sigma_{\theta\theta}\varepsilon_{\theta\theta} + \sigma_{\phi\phi}\varepsilon_{\phi\phi})(x+a)^2 \sin\theta dx d\theta d\phi, \quad (8)$$

where the nonvanishing components of the strain are

$$\varepsilon_{\theta\theta} = \frac{1}{a}\left(\frac{\partial u}{\partial \theta} + w\right) + \frac{x}{a^2}\left(\frac{\partial u}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right), \quad (9)$$

and

$$\varepsilon_{\phi\phi} = \frac{1}{a}(\cot\theta u + w) + \frac{x}{a^2}\cot\theta\left(u - \frac{\partial w}{\partial \theta}\right), \quad (10)$$

and where the nonzero stress components are

$$\sigma_{\theta\theta} = \frac{E}{1-\nu^2}(\varepsilon_{\theta\theta} + \nu\varepsilon_{\phi\phi}), \quad (11)$$

and

$$\sigma_{\phi\phi} = \frac{E}{1-\nu^2}(\varepsilon_{\phi\phi} + \nu\varepsilon_{\theta\theta}), \quad (12)$$

where E is Young's modulus. By substitution the potential energy becomes

$$V = \frac{1}{2}\int_0^{\pi}\int_0^{2\pi}\int_{-h/2}^{h/2}\left[\frac{E}{1-\nu^2}\frac{1}{(x+a)^2}\left([(1+\frac{x}{a})\frac{\partial u}{\partial \theta} - \frac{x}{a}\frac{\partial^2 w}{\partial \theta^2} + w^2] + \{\cot\theta[(1+\frac{x}{a})u - \frac{x}{a}\frac{\partial w}{\partial \theta}] + w\}^2\right.\right.$$
$$\left.\left. + 2\nu\{\cot\theta[(1+\frac{x}{a})u - \frac{x}{a}\frac{\partial w}{\partial \theta}] + w\}[(1+\frac{x}{a})\frac{\partial u}{\partial \theta} - \frac{x}{a}\frac{\partial^2 w}{\partial \theta^2} + w]\right)\right](x+a)^2\sin\theta dx d\theta, \quad (13)$$

which after integration is

$$V = \frac{\pi E h}{1-\nu^2}\int_0^{\pi}\{(w + \frac{\partial u}{\partial \theta})^2 + (w + u\cot\theta)^2 + 2\nu(w + \frac{\partial u}{\partial \theta})(w + u\cot\theta)$$
$$+\beta^2[(\frac{\partial u}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2})^2\cot^2\theta(u - \frac{\partial w}{\partial \theta})^2 + 2\nu\cot\theta(u - \frac{\partial w}{\partial \theta})(\frac{\partial u}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2})]\}\sin\theta d\theta. \quad (14)$$

Terms in the potential energy proportional to β^2 are due to bending stresses.

And finally, the work done by the pressure of the surrounding fluid on the spherical shell is given by

$$W = 2\pi a^2 \int_0^{\pi} p_a w \sin\theta d\theta, \tag{15}$$

where p_a is the pressure at the surface.

2.2. The Lagrangian density and its equations of motion

Integration along the polar angle θ is intrinsic to the problem, therefore we must turn to a *Lagrangian density* formulation to solve for the equations of motion. Our Lagrangian density is just

$$\begin{aligned} \mathrm{L} = {} & \pi\rho_s h a^2[(1+6\beta^2+1.8\beta^4)\dot{u}^2 - (6\beta^2+3.6\beta^4)\dot{u}\frac{\partial\dot{w}}{\partial\theta} + (\beta^2+1.8\beta^4)(\frac{\partial\dot{w}}{\partial\theta})^2 + (1+\beta^2)\dot{w}^2]\sin\theta \\ & - \frac{\pi E h}{1-\nu^2}[(w+\frac{\partial u}{\partial\theta})^2 + (w+u\cot\theta)^2 + 2\nu(w+\frac{\partial u}{\partial\theta})(w+u\cot\theta) \\ & + \beta^2[(\frac{\partial u}{\partial\theta} - \frac{\partial^2 w}{\partial\theta^2})^2 + \cot^2\theta(u-\frac{\partial w}{\partial\theta})^2 + 2\nu\cot\theta(u-\frac{\partial w}{\partial\theta})(\frac{\partial u}{\partial\theta} - \frac{\partial^2 w}{\partial\theta^2})]]\sin\theta + 2\pi a^2 p_a w\sin\theta, \end{aligned} \tag{16}$$

with corresponding differential equations

$$0 = \frac{\partial \mathrm{L}}{\partial u} - \frac{d}{d\theta}\frac{\partial \mathrm{L}}{\partial u_\theta} - \frac{d}{dt}\frac{\partial \mathrm{L}}{\partial u_t}, \tag{17}$$

and

$$0 = \frac{\partial \mathrm{L}}{\partial w} - \frac{d}{d\theta}\frac{\partial \mathrm{L}}{\partial w_\theta} - \frac{d}{dt}\frac{\partial \mathrm{L}}{\partial w_t} + \frac{d^2}{d\theta dt}\frac{\partial \mathrm{L}}{\partial w_{\theta t}} + \frac{d^2}{d\theta^2}\frac{\partial \mathrm{L}}{\partial w_{\theta\theta}}, \tag{18}$$

where subscripts denote differentiation of the variable with respect to the subscript.

By substitution of Eqs. (17) and (18) into (16) we obtain

$$\begin{aligned} 0 = {} & (1+\beta^2)\left[\frac{\partial^2 u}{\partial\theta^2} + \cot\theta\frac{\partial u}{\partial\theta} - (\nu+\cot^2\theta)u\right] - \beta^2\frac{\partial^3 w}{\partial\theta^3} - \beta^2\cot\theta\frac{\partial^2 w}{\partial\theta^2} + [(1+\nu)+\beta^2(\nu+\cot^2\theta)]\frac{\partial w}{\partial\theta} \\ & - \frac{a^2}{c_p^2}[(1.8\beta^4+6\beta^2+1)\frac{\partial^2 u}{\partial t^2} - (1.8\beta^4+3\beta^2)\frac{\partial^3 w}{\partial\theta\partial t^2}], \end{aligned} \tag{19}$$

and

$$\begin{aligned} -p_a\frac{(1-\nu^2)a^2}{Eh} = {} & \beta^2\frac{\partial^3 u}{\partial\theta^3} + 2\beta^2\cot\theta\frac{\partial^2 u}{\partial\theta^2} - [(1+\nu)(1+\beta^2)+\beta^2\cot^2\theta)]\frac{\partial u}{\partial\theta} \\ & + \cot\theta[(2-\nu+\cot^2\theta)\beta^2 - (1+\nu)]u \\ & - \beta^2\frac{\partial^4 w}{\partial\theta^4} - 2\beta^2\cot\theta\frac{\partial^3 w}{\partial\theta^3} + \beta^2(1+\nu+\cot^2\theta)\frac{\partial^2 w}{\partial\theta^2} - \beta^2\cot\theta(2-\nu+\cot^2\theta)\frac{\partial w}{\partial\theta} - 2(1+\nu)w \\ & + \frac{a^2}{c_p^2}[-(1.8\beta^4+3\beta^2)\frac{\partial^3 u}{\partial\theta\partial t^2} - (1.8\beta^4+3\beta^2)\cot\theta\frac{\partial^2 u}{\partial t^2} \\ & + (1.8\beta^4+\beta^2)\frac{\partial^4 w}{\partial\theta^2\partial t^2} + (1.8\beta^4+\beta^2)\frac{\partial^3 w}{\partial\theta\partial t^2}\cot\theta - (\beta^2+1)\frac{\partial^2 w}{\partial t^2}. \end{aligned} \tag{20}$$

These differential equations of motion (19) and (20) have solutions of the form

$$u(\eta) = \sum_{n=0}^{\infty} U_n (1-\eta^2)^{1/2} \frac{dP_n}{d\eta}, \tag{21}$$

and

$$w(\eta) = \sum_{n=0}^{\infty} W_n P_n(\eta), \tag{22}$$

where $\eta = \cos\theta$ and $P_n(\eta)$ are the Legendre polynomials of the first kind of order n. When the differential equations of motion (19) and (20) are expanded in terms of Eqs. (21) and (22), we obtain a set of linear equations in terms of U_n and W_n, whose determinant must vanish. We consider two cases: with and without fluid loading.

2.3. Vacuum case

The simpler case is that when the spherical shell is surrounded by a vacuum such that there is no damping. In this case, the pressure at the surface vanishes: $p_a = 0$. The set of linear equations the expansion coefficients must satisfy are

$$0 = [\Omega^2(1+6\beta^2+1.8\beta^4) - (1+\beta^2)\kappa]U_n + [\Omega^2(3\beta^2+1.8\beta^4) - \beta^2\kappa - (1+\nu)]W_n, \tag{23}$$

and

$$0 = -\lambda_n[(\kappa-3)\beta^2 - 1.8\beta^4 + 1 + \nu]U_n + [\Omega^2(1+2\beta^2+1.8\beta^4) - 2(1+\nu) - \beta^2\kappa\lambda_n]W_n, \tag{24}$$

where $\Omega = \omega a / c_p$, $\kappa = \nu + \lambda_n - 1$, and $\lambda_n = n(n+1)$. In order for Eqs. (23) and (24) to be satisfied simultaneously with a non-trivial solution the determinant of the system must vanish:

$$\begin{aligned} 0 = {} & \Omega^4(1+6\beta^2+1.8\beta^4)(1+2\beta^2+1.8\beta^4) + \Omega^2\{(3\beta^2+1.8\beta^4)\lambda_n[(\kappa-3)\beta^2 - 1.8\beta^4 + 1 + \nu] \\ & -[2(1+\nu)+\beta^2\kappa\lambda_n](1+6\beta^2+1.8\beta^4) - (1+\beta^2)\kappa(1+2\beta^2+1.8\beta^4)\} \\ & +(1+\beta^2)\kappa[2(1+\nu)+\beta^2\kappa\lambda_n] - \lambda_n[(\kappa-3)\beta^2 - 1.8\beta^4 + 1 + \nu](\beta^2\kappa + 1 + \nu). \end{aligned} \tag{25}$$

Since there are no damping terms, the shell vibrates, in theory, forever. Thus, the normalized frequency Ω can be taken to be real. Equation (25) is quadratic in Ω^2, thus we expect two real roots to (25) and thus two modes for the motion of the shell. They are the symmetric and antisymmetric modes.

2.4. Fluid loaded case

For the fluid loaded case, we must consider a modal expansion of the surface pressure in terms of the specific acoustic impedance z_n. In its most general form this is

$$p(a,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} z_n \dot{W}_{mn} P_n^m(\cos\theta)\cos m\phi, \tag{26}$$

where

$$z_n = i\rho c \frac{h_n(ka)}{h_n'(ka)}. \tag{27}$$

The specific acoustic impedance z_n can be split into real and imaginary parts:

$$z_n = r_n - i\omega m_n, \tag{28}$$

where

$$r_n = \rho c \mathrm{Re}\left\{\frac{ih_n(ka)}{h_n'(ka)}\right\}, \tag{29}$$

and

$$m_n = -\frac{\rho c}{\omega} \mathrm{Im}\left\{\frac{ih_n(ka)}{h_n'(ka)}\right\}. \tag{30}$$

For the case we are considering of axisymmetric motion, the surface pressure is given by

$$p_a(\theta) = -\sum_{n=0}^{\infty} z_n \dot{W}_n P_n(\cos\theta), \tag{31}$$

or

$$p_a(\theta) = -\sum_{n=0}^{\infty} (-i\omega W_n r_n - \omega^2 W_n m_n) P_n(\cos\theta). \tag{32}$$

Use of Eq. (32) in our set of differential equations of motion (19) and (20) yields the following set of linear equations for the expansion coefficients in the case of a fluid loaded spherical shell:

$$0 = [\Omega^2(1+6\beta^2+1.8\beta^4) - (1+\beta^2)\kappa]U_n + [\Omega^2(3\beta^2+1.8\beta^4) - \beta^2\kappa - (1+\nu)]W_n, \tag{33}$$

and

$$0 = -\lambda_n[(\kappa-3)\beta^2 - 1.8\beta^4 + 1 + \nu]U_n + [\Omega^2(1+\alpha+2\beta^2+1.8\beta^4) - 2(1+\nu) + \Omega i\gamma - \beta^2\kappa\lambda_n]W_n, \tag{34}$$

where

$$\alpha = \frac{m_n}{\rho_s h}, \tag{35}$$

and

$$\gamma = \frac{a}{h}\frac{r_n}{\rho_s c_p}. \tag{36}$$

Again the determinant of Eqs. (33) and (34) must vanish. However, in this instance the value of Ω must be taken to be complex; the resonances have a width that depends on the damping. The result of setting this determinant to zero is

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$$0 = \Omega^4(1+6\beta^2+1.8\beta^4)(1+\alpha+2\beta^2+1.8\beta^4)+\Omega^3 i\gamma(1+6\beta^2+1.8\beta^4)$$
$$+\Omega^2\{(3\beta^2+1.8\beta^4)\lambda_n[(\kappa-3)\beta^2-1.8\beta^4+1+\nu]$$
$$-[2(1+\nu)+\beta^2\kappa\lambda_n](1+6\beta^2+1.8\beta^4)-(1+\beta^2)\kappa(1+\alpha+2\beta^2+1.8\beta^4)\}$$
$$+\Omega[-i\gamma(1+\beta^2)\kappa]+(1+\beta^2)\kappa[2(1+\nu)+\beta^2\kappa\lambda_n]-\lambda_n[(\kappa-3)\beta^2-1.8\beta^4+1+\nu](\beta^2\kappa+1+\nu). \quad (3$$

Equation (37) has four complex roots. From work with an exact modal solution to the problem, we exp two roots to be associated with the symmetric and antisymmetric modes of the shell. We expect the other t roots to be associated with a water-borne pseudo-Stoneley wave.

3. CONCLUSIONS

The next step is to plot the roots of Eqs. (25) and (37) to compare the resonances predicted by the models with those given by exact modal expansion solutions. By suppressing α and γ, the model associat with Eq. (37) reverts to the vacuum case model associated with Eq. (25). Similarly suppression of factors β in Eq. (25) will result in a reversion to a previously derived solution [1]. In a previous paper[2] we ranked t three different models according to their degree of physicality and compared their results for various relati shell thicknesses against each other and against the exact results of the modal expansion model. We al considered the limitations of each of the models including the exact solution, as well as those of shell mode in general.

These models, fluid loaded, vacuo case, and membrane, were successively less physically sophisticate and gave successively less good comparison with the exact (modal expansion) results. The interested reader referred to that earlier work for the comparisons of the models. We shall content ourselves with showing th results of our best model , that corresponding to the roots of Eq. (37). In Fig. 2 we see the thin spherical ste shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by this model whic includes proper fluid loading. Here and in the succeeding figures thick means $h/a = 0.1$; thin means h/a = 0.01. The shell material is a generic steel with density $\rho_s = 7.7$ times that of water, shear velocity $v_s = 3.2$ km/s, and longitudinal velocity $v_l = 5.95$ km/s. The surrounding fluid is taken to be water with density ρ = 1000 kg/m^3 and sound velocity $c_w = 1.4825$ km/s. The exact and shell theory calculations agree well for th dilatational (symmetric) resonances and exhibit a marked improvement over the results of our membrane [Eq. (25)] and vacuum models [Eq. (37) with α and γ suppressed] for the first several shell theory symmetric mode resonances. This is due to the inclusion of both rotary inertia and fluid loading in the model. The flexural (antisymmetric) mode resonances show the appropriate behavior on this rather limited size parameter scale. Next, in Fig. 3 we have a plot of thick spherical steel shell dilatational (symmetric) and flexural (antisymmetric) mode resonances calculated by shell theory with fluid loading. As in the vacuum case as well as for the membrane model, the dilatational (symmetric) mode resonances compare well for exact and shell theory methods. Finally in Fig. 4 we show a comparison of modal resonance predictions for a 1% thick steel shell (thin shell) for the low size parameter (= ka, where k is wavenumber and a the radius of the midshell) region, just where one would expect to see pseudo-Stoneley resonances, if the model is capable of predicting them. The models compared are our vacuum model, our fluid loaded model, and an exact calculation of the pseudo-Stoneley resonances in this region. As expected, the vacuum model misses the water-borne waves completely, but our fluid loaded model does predict them, and predicts them exactly.

For future reference, the flexural (antisymmetric) mode resonances, as calculated by our shell theory with fluid loading, do not have the correct asymptotic limit for large size parameter, although they do exhibit the correct behavior for lower values of ka.. The proper correction for the large ka asymptotic resonance behavior could be most easily found by including the shear stress distortion along with a Timoshenko-Mindlin[3,4] shape factor. Inclusion of the shear distortion in the potential energy would make the flexural modes asymptote to the coincidence velocity and the shape factor can be adjusted so that they asymptote to the Rayleigh velocity as expected. Work is now under way to do just this.

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